

Math 2301 Final Exam

Fall 2021-22

Name: _____

Instructor: _____

There are 16 questions on this exam on 8 pages (not counting this coverpage). Answer each question in the space provided.

Question	Points	Score
1	12	
2	4	
3	4	
4	6	
5	4	
6	10	
7	5	
8	5	
9	10	
10	8	
11	5	
12	16	
13	8	
14	4	
15	5	
16	4	
Total:	110	

1. Differentiate the functions:

(a) (6 points) $f(x) = \frac{\ln(x)}{1+2x}$.

(b) (6 points) $f(z) = (\cos(z) + 3)(6z - 5z^9)$.

2. (4 points) If $f'(2) = 5$, circle ALL the following that **must** be true.

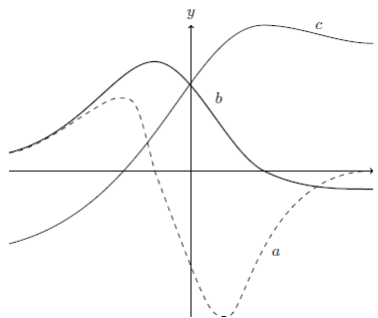
- (a) f is continuous at $x = 2$. (b) $f(2) = 5$. (c) f is differentiable at $x = 2$.
(d) The slope of the tangent line of f at $x = 2$ is 5. (e) The rate of change of f at $x = 2$ is 5.

3. (4 points) If $\lim_{x \rightarrow a^-} f(x) = L = \lim_{x \rightarrow a^+} f(x)$, where L is a real number, circle ALL the following that **must** be true.

- (a) f is continuous at $x = a$. (b) f is defined at $x = a$. (c) $\lim_{x \rightarrow a} f(x) = L$.
(d) $f(a) = L$. (e) f is differentiable at $x = a$.

4. (6 points) The figure shows the graphs of f , f' , and f'' . Identify each curve (fill in the blank with your choice of a , b , or c for each).

f is _____
 f' is _____
 f'' is _____



5. (4 points) Let $f(x) = x^2 + 3x - 1$ on the interval $[-2, 2]$. Find a value c in $[a, b]$ guaranteed by the Mean Value Theorem.
6. (10 points) In this question, distance is measured in feet (ft), and time is measured in seconds (s). Provide correct units in your answers.
- (a) If a particle has the velocity function $v(t) = 3t - 6$ ft/s, $0 \leq t \leq 3$:
- Find the distance traveled by the particle.
 - Find the displacement of the particle.
- (b) At what time will an object be at rest if its position function is given by $p(t) = t^3 + 9t^2 + 24t$ ft?
- (c) An objects position is given by $p(t) = t^4 + 4t^2$ ft. What is the objects acceleration at $t = 3$?

7. (5 points) Find $\frac{dy}{dx}$ using implicit differentiation.

$$\sin(y) + y^3 = 6 - x^3.$$

8. (5 points) (a) Find the equation of the tangent line to the graph of

$$f(x) = \frac{10}{x+1}$$

at the point $x = 9$.

- (b) Use your answer in part (a) to approximate $f(9.1)$.

9. (10 points) Sketch the graph of a function $f(x)$ that satisfies all of the following:

1. f is continuous on the intervals $(-\infty, 1)$, $(-1, 2)$, and $(2, \infty)$.
2. $f(-1) = 3$
3. $\lim_{x \rightarrow -1} f(x) = -2$
4. $f(0) = 1$
5. $f'(0) = 0$
6. $\lim_{x \rightarrow 1} f(x) = f(1)$.
7. $f'(1)$ does not exist.
8. $\lim_{x \rightarrow -\infty} f(x) = \infty$.
9. $\lim_{x \rightarrow 2^+} f(x) = -\infty$.

10. (8 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

11. (5 points) Use the graph of $f(x)$ to find the values of the following integrals. The first area (with full shading) is 8 square units, the second area (with slanted shading) is 21 square units.

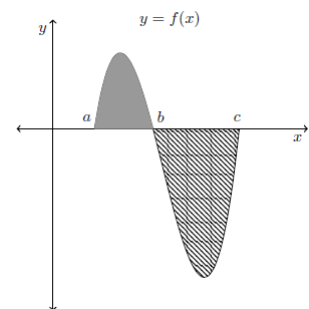
(a) $\int_a^b f(x)dx =$

(b) $\int_b^c f(x)dx =$

(c) $\int_a^c f(x)dx =$

(d) $\int_a^c |f(x)|dx =$

- (e) If $a = 1$ and $c = 7$. What is the average value of f on the interval $[1, 7]$?



12. Evaluate the integrals:

(a) (4 points)

$$\int_1^2 x^2 e^{1-x^3} dx.$$

(b) (4 points)

$$\int_0^\pi (\sqrt{x} - \cos(x)) dx.$$

(c) (4 points)

$$\int (2x^4 + x^{-2} - 8) dx.$$

(d) (4 points)

$$\int 5x \cos(2x^2 - 5) dx.$$

13. (8 points) A 10ft ladder is leaning against a house while the base is pulled away at a constant rate of 1ft/s. At what rate is the top of the ladder sliding down the side of the house when the base is 6 feet from the house?

14. (4 points) Let

$$g(x) = \int_1^x \sqrt{t^3 + 4t} \, dt.$$

Calculate $g'(2)$.

15. (5 points) The function $f(x)$ (sketched below) is a twice differentiable function. Answer True (T) or False (F) to each of the following statements.

- (a) $f(1) < f'(1)$
- (b) $f'(1) < f''(1)$
- (c) The point $(1, f(1))$ is an inflection point.
- (d) If a left Riemann approximation (with $n = 4$) is used to approximate $\int_1^2 f(x) dx$, it would be an *under* approximation.
- (e) If Newton's Method is used to locate a root of the equation $f(x) = 0$ with the initial approximation of $x_1 = 1.5$, then the second approximation, x_2 , would be to the left of 1.



16. (4 points) Answer True (T) or False (F) to each of the following statements.

- (a) If f has a discontinuity at $x = 3$, then $\lim_{x \rightarrow 3} f(x)$ does not exist.
- (b) The function $f(x)$ is differentiable everywhere on the real line. If $x = x_0$ is the only critical value of $f(x)$ with $f'(x_0) = 0$, then $f(x_0)$ must be either the global maximum or the global minimum.
- (c) L'Hôpital's rule can be used to find

$$\lim_{x \rightarrow 0} \frac{x^7}{e^x}.$$

- (d) L'Hôpital's rule can be used to find

$$\lim_{x \rightarrow \infty} \frac{9 - 4x}{x + 1}.$$